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# Effect of a tilted magnetic field on the orientation of Wigner crystals

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**Abstract**

We study the effect of a tilted magnetic field on the orientation of Wigner crystals by taking account of the width of a quantum well in the $z$-direction. It is found that the cohesive energy of the electronic crystal is always lower for the [110] direction parallel to the in-plane field. In a realistic sample, a domain structure forms in the electronic solid and each domain orients randomly when the magnetic field is normal to the quantum well. As the field is tilted by an angle, the electronic crystal favours alignment along a preferred direction which is determined by the in-plane magnetic field. The orientation stabilization is strengthened for wider quantum wells as well as for larger tilt angles. The possible consequence of the tilted field for the transport property in the electronic solid is discussed.

## 1. Introduction

It was initially predicted by Wigner that two-dimensional (2D) electrons crystallize into a triangular lattice in the low-density limit where the electron–electron interactions dominate over the kinetic energy. In an ideally clean 2D system, the critical $r_s$ ($r_s = U/\epsilon_F$, corresponding to the ratio of the Coulomb energy scale $U$ to the kinetic energy scale of the Fermi energy $\epsilon_F$) was presented to be $37 \pm 5$ from quantum Monte Carlo simulations [1]. A strong magnetic field perpendicular to the 2D plane can effectively localize electron wavefunctions while keeping the kinetic energy controlled [2]. Since this lessens the otherwise severe low-density condition, it is believed that the Wigner crystal (WC) can be stabilized in a sufficiently strong magnetic field [3–5]. Approximate calculations [6] have shown that the WC becomes the lowest-energy state when the filling factor $\nu < 1/6$ for the GaAs/AlGaAs electron system and around $\nu = 1/3$ for the hole system. Since the impurities pin the electronic crystal, a domain structure forms in a realistic sample [7]. While the electrons in a domain have an order as they are in the ideal crystal, the orientations of the domains are random.

Currently, the measurement in a tilted field has become an established technique to explore the various correlated properties in single-layer as well as in double-layer

  

2D electron systems [8]. In a previous work [9], we have compared the ground state energies of the generalized Laughlin liquid [10] to the electronic solid state at a given tilt angle. It was found that the critical filling factor $\nu_c$ at the solid–liquid transition increases with increasing tilt angle.

In this work, we will examine the relation of the orientation of the hexagonal WC to the in-plane magnetic field as well as the width of the 2D quantum well. In a wide quantum well, the electron wavefunction extends in the $z$-direction, hence may reduce the Coulomb interactions. The in-plane field deforms the electron wavefunction, causing the interaction energy to vary according to the different patterns or orientations of the electronic crystals. We calculate the cohesive energy of the electronic crystal in a Hartree–Fock (HF) approximation. We find that it becomes anisotropic in the tilted magnetic field. The [110] axis of the hexagonal electronic crystal favours alignment along the direction of the in-plane magnetic field. This trend of orientation stabilization is strengthened for larger tilt angles. It also shows that the energy difference between two orthogonal orientations of the electronic crystal increases with the width of the quantum well. The in-plane field favours the domains orienting to the same direction. Thus, the effective impurity density is reduced as the field is tilted. We will discuss the possible consequence of such an effect on the transport properties in the electronic solids.

## 2. Anisotropic cohesive energy of the electronic crystal

Consider an electron moving on an $x$–$y$ plane under the influence of a strong magnetic field which is tilted by an angle $\theta$ to the normal, with $\vec{B} = (B\tan\theta, 0, B)$. The electron is confined in a harmonic potential $V(z) = \frac{1}{2}m_b\Omega^2 z^2$ in the $z$-direction, where $m_b$ is the band mass of the electron and $\Omega$ the characteristic frequency. Such a quantum well has been chosen to deal with many quantum Hall systems [11, 12] to substitute the realistic potential which is either triangular or square. It was also used to discuss the giant magneto-resistance induced by a parallel magnetic field [13]. We work in the 'Landau gauge' by choosing the vector potential $\vec{A} = \{0, xB_z - zB_x, 0\}$. The single-particle wavefunctions for the lowest LLs are

$$\phi_X(\vec{r}) = \frac{1}{\sqrt{L_y}} e^{-iXy/l_B^2} \Phi_0^{\omega_+}\left(\frac{-(x-X)\sin\tilde{\theta} + z\cos\tilde{\theta}}{l_+}\right)\Phi_0^{\omega_-}\left(\frac{(x-X)\cos\tilde{\theta} + z\sin\tilde{\theta}}{l_-}\right), \quad (1)$$

where $l_B$ is the magnetic length and $l_\pm^2 = \hbar/m\omega_\pm$. $X$ is an integer multiple of $2\pi l_B^2/L_y$. $\Phi_0^{\omega_\pm}$ is the harmonic oscillator wavefunction in the lowest energy level corresponding to the frequencies $\omega_\pm$ and $\tan\tilde{\theta} = \frac{\omega_c^2}{\omega_z^2 - \omega_c^2}\tan\theta$, with the cyclotron frequency $\omega_c = eB/m_b c$. The frequencies $\omega_\pm$ are given by [10]

$$\omega_\pm^2 = \frac{1}{2}\left(\Omega^2 + \frac{\omega_c^2}{\cos^2\theta}\right) \pm \sqrt{\frac{1}{4}\left(\Omega^2 - \frac{\omega_c^2}{\cos^2\theta}\right)^2 + \Omega^2\omega_c^2\tan^2\theta}. \quad (2)$$

The Hamiltonian is given by

$$\hat{H} = \frac{1}{2L_xL_y}\sum_{\vec{q}} \hat{\rho}(\vec{q})v(\vec{q})\hat{\rho}(-\vec{q}), \quad (3)$$

where $v(\vec{q}) = \frac{4\pi e^2}{\kappa_0(\vec{q}_\parallel^2 + q_z^2)}$ is the Fourier transformation of the Coulomb interaction. Here $\vec{q}_\parallel$ is the in-plane momentum and $q_z$ is the momentum perpendicular to the quantum well.

From equation (1), the electron density operator is expressed in the momentum space as

$$\hat{\rho}(\vec{q}) = \sum_X e^{iq_xX} a_{X_-}^\dagger a_{X_+} F^\theta(\vec{q}), \quad (4)$$

where $X_\pm = X \pm q_y l_B^2/2$. $a_X^\dagger$ ($a_X$) creates (destroys) an electron in the state $\phi_X$. Here $F^\theta(\vec{q}) = e^{-\gamma^2/4-\alpha^2/4}$, with

$$\begin{aligned}\alpha^2 &= (q_x\cos\tilde{\theta} - q_z\sin\tilde{\theta})^2 l_-^2 + q_y^2 l_B^4/l_-^2\cos^2\tilde{\theta}\\ \gamma^2 &= (q_z\cos\tilde{\theta} + q_x\sin\tilde{\theta})^2 l_+^2 + q_y^2 l_B^4/l_+^2\sin^2\tilde{\theta}.\end{aligned} \tag{5}$$

Substitute equation (4) into (3) and carry out the usual procedure of the HF decoupling of the Hamiltonian; we get

$$H_{HF} = \frac{n_L}{2}\sum_{\vec{q}_\parallel} u_{HF}(\vec{q}_\parallel)\Delta(-\vec{q}_\parallel)\sum_X e^{-iq_x X} a_{X_+}^\dagger a_{X_-}, \tag{6}$$

where $n_L = 1/2\pi l_B^2$ is the density of one completely filled LL and

$$\Delta(\vec{q}_\parallel) = \frac{2\pi l_B^2}{L_x L_y}\sum_X e^{-iq_x X}\langle a_{X_+}^\dagger a_{X_-}\rangle \tag{7}$$

is the order parameter of the charge density wave (CDW). The HF potential is denoted with $u_{HF}(\vec{q}_\parallel) = u_H(\vec{q}_\parallel) - u_{ex}(\vec{q}_\parallel)$. The Hartree term $u_H(\vec{q}_\parallel)$ is given by (in units of $e^2/\kappa_0 l_B$)

$$u_H(\vec{q}_\parallel) = \int \frac{dq_z}{\pi l_B}\frac{1}{\vec{q}_\parallel^2 + q_z^2}[F^\theta(\vec{q})]^2, \tag{8}$$

and the exchange term $u_{ex}(\vec{q}_\parallel)$ in the reciprocal space turns out to be proportional to the real-space Hartree potential as [11, 14]

$$u_{ex}(\vec{q}_\parallel) = -2\pi l_B^2\int \frac{d\vec{p}_\parallel}{(2\pi)^2}u_H(\vec{p}_\parallel)e^{i\vec{p}_\parallel\times\vec{q}_\parallel l_B^2}. \tag{9}$$

Allowing the charge density wave by making the ansatz in the plane

$$\langle a^\dagger_{X-Q_y l_B^2/2} a_{X+Q_y l_B^2/2}\rangle = e^{iQ_x X}\Delta(\vec{Q}), \tag{10}$$

where $\Delta(\vec{Q})$ is the order parameter, the cohesive energy can be calculated in the same way as has been done in [2, 14, 15]:

$$E_{coh} = \frac{1}{2\nu}\sum_{\vec{Q}\neq 0} u_{HF}(\vec{Q})|\Delta(\vec{Q})|^2, \tag{11}$$

where $\nu$ is the filling factor of the lowest Landau level.

We carry out the self-consistent HF computation on a hexagonal lattice with the wavevectors of the order parameters as $\vec{Q} = \left[(j+\frac{1}{2})Q_0, \frac{\sqrt{3}}{2}kQ_0\right]$, where $j$ and $k$ are integers. Following the procedure in [2], when $NQ_x^0 Q_y^0 l_B^2 = 2M\pi$, with $N$ and $M$ being integers, the Landau level splits into $N$ Hofstadter bands. When $N = 6$ and $M = 1$ the WC has the lowest energy. In our calculations, we choose $\nu = 0.12$, at which the ground state is a Wigner crystal.

Figure 1 displays the dependence of the cohesive energy of the electronic crystal on the tilt angle $\theta$ for various orientation angles of the crystal to the in-plane magnetic field, where $\phi$ is the angle between one side of the hexagonal lattice and the in-plane field. It shows that of the two typical configurations of orientation with respect to the in-plane field, the [100] and the [110], the energy is always lower for the [110] direction parallel to the in-plane field. The energy difference increases with the tilt angle. In figure 2 we plot the relation of the cohesive energy of the hexagonal lattice to the characteristic frequency $\Omega$ for tilt angles $\theta = 0^\circ$ and $43.2^\circ$, respectively. The [110] direction is along the in-plane field for both curves. The smaller the characteristic frequency is, i.e., the wider the quantum well is, the higher the cohesive energy. The in-plane magnetic field can lower the cohesive energy of the electronic crystal. From figure 2 one can see that the energy difference becomes larger for wider quantum wells.

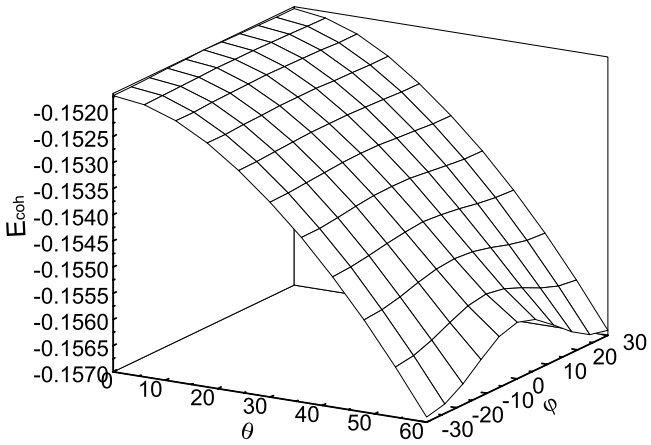


**Figure 1.** A 3D graph of the cohesive energy of the WC with respect to the tilt angle $\theta$ as well as the orientation angle $\phi$.

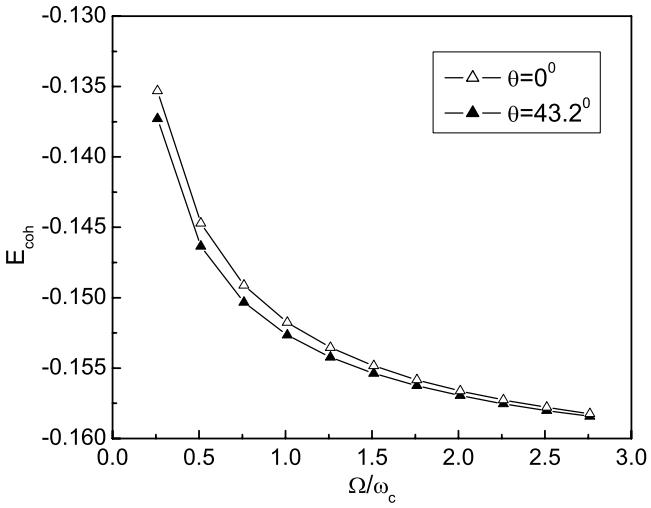


**Figure 2.** The cohesive energy of the hexagonal lattice with the characteristic frequency $\Omega$ for tilt angle $\theta = 0°$ and 43.2°, respectively. The energy difference increases with decreased $\Omega$.

Our calculations show that when the tilt angle increases further, the energy difference will increase significantly, implying that the in-plane magnetic field stabilizes the orientation of the electronic crystal more effectively.

## 3. Transport property of the electronic solid

Pinning of the Wigner crystal by impurities as a result of breaking of the translational invariance has been widely investigated [4, 16]. Sherman [7] had studied the angular pinning and the domain structure of the electronic crystal mediated by acoustic-phonon in III–V semiconductor. Our calculation shows that the in-plane magnetic field may serve as a tunable means to probe the orientation of the crystal. Below we will explore the implications of the preferred orientation of the electronic crystal to the transport measurements. In realistic samples a domain structure

is formed due to a finite impurity density. The electrons in each domain are ordered as they are in the crystal. In the absence of the in-plane field, each domain orients randomly, just like the domains in ferromagnets. An ideal electronic crystal is an insulator and the conductivity $\sigma_{xx} \propto \mathrm{e}^{-\Delta_0/k_\mathrm{B}T}$ [17, 18]. This thermal activation form of the conductivity implies that the electrons are hopping with a fixed-range mechanism. It has been confirmed by experiments that $\Delta_0$ is of the order of 1 K [19]. In a realistic domain structure, however, the electrons may hop between the edges of the randomly oriented domains. Since the experimentally reachable temperature may be as low as 10 mK, the variable-range hopping mechanism may work in this temperature regime [20]. In the following, we will discuss the possible consequence of the tilted field on the transport properties.

In the usual Anderson localization the envelope of the wavefunction falls off exponentially as $\phi_0 \sim \mathrm{e}^{-r/\xi}$, where $\xi$ is the localization length. With a magnetic field the electronic wavefunction of a perfect system is essentially a Gaussian as $\phi_\mathrm{m} \sim \mathrm{e}^{-r^2/2l_\mathrm{B}^2}$. In a slightly disordered system one can think that some of the states will be pinned at certain isolated impurity sites. The mixing of these states due to quantum mechanical tunnelling leads to a simple exponential tail in the wavefunction [18]. In a strong magnetic field, the electrons condense into a crystal at lower filling factors. When the temperature is high enough the transport is of the thermal activation form, which implies that the electrons are hopping with a fixed-range mechanism [17, 18]. The hopping range is determined by $R_0 = \sqrt{1/\pi n_\mathrm{I}}$, where $n_\mathrm{I}$ is the impurity density. However, localized states may exist along the edges of the domains of the electronic crystal because of the impurities. When the temperature is sufficiently low that there is nearly no phonon with energy to assist the electron making the nearest hopping, Mott's variable-hopping mechanism [20] allows the electrons to hop a larger distance $R > R_0$ to a state which has a smaller energy difference $\Delta(R)$. In turn, the hopping conduction is determined by the typical decay rate of the tails of the wavefunction. The hopping probability is then given by

$$p \propto \exp[-R/\xi - \Delta/k_\mathrm{B}T], \tag{12}$$

where $R = |\vec{R}_i - \vec{R}_j|$ and $\Delta$ is the activation energy.

As in the quantum Hall effect regime, strong interaction between electrons leads to the system condensing into a WC. The Coulomb gap depresses the density of states near the Fermi surface [21, 22]. Efros *et al* [23] had derived the density of states near the Fermi surface $N(E) \propto |\Delta E| = |E - E_\mathrm{F}|$. From these considerations, one can get the conductivity in the variable-range hopping as [21]

$$\sigma_{xx} \propto p \propto \mathrm{e}^{-A/T^{1/2}}, \tag{13}$$

where $A = [\frac{4\hbar v_\mathrm{F}}{k_\mathrm{B}\xi}]^{1/2}$. The characteristic temperature $T_0$ above which the fixed-range hopping dominates is determined by $\bar{R} = R_0$, namely

$$k_\mathrm{B}T_0 = \frac{2\hbar^2 l_\mathrm{B}}{m_\mathrm{b}}(\pi n_\mathrm{I})^{3/2}. \tag{14}$$

Now, we discuss the possible effect of the tilted field. As we have discussed, the existence of an in-plane field deforms the electron wavefunction. However, this wavefunction deformation does not qualitatively change the electron hopping mechanism at a given temperature. The major effect of the tilted field would be on the variation of $T_0$. As we have shown, the in-plane field lowers the cohesive energy of the Wigner crystal and forces the domains to align to the same direction. Thus, the role of the in-plane field is to integrate the domains into larger ones. In this way, the in-plane field causes some of impurities to be irrelevant and therefore reduces the effective impurity density. To determine $T_0$ from (14), only

the relevant impurities should be counted in. Hence, one can replace $n_I$ by an effective impurity density $n_I(B_\parallel)$. From equation (14), we see that $T_0$ is sensitive to $n_I(B_\parallel)$. In a strong magnetic field the decay length is comparable to the cyclotron radius $\xi \sim R_c$ [24]. For a sample with $n_I \sim 1.0 \times 10^8\,\mathrm{cm}^{-2}$, we estimate $T_0 \sim 40$ mK. This temperature is experimentally reachable. Therefore, it is possible to observe a change of transport behaviour that displays a crossover from the variable- to the fixed-range hopping under proper parameters and temperature as the tilt angle rises.

## 4. Summary

We have shown that the WC has a preferred orientation with respect to the in-plane magnetic field. We argued that there are domains in a realistic sample and predicted that the temperature dependence of the transport behaviour may be different in a different temperature regime. Moreover, we emphasized that this preferred orientation of the crystal may lead to an in-plane field induced crossover from the variable-range hopping to the fixed-range hopping of the transport mechanism in the 2D electronic solid. We expect future experiments to verify our prediction.

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